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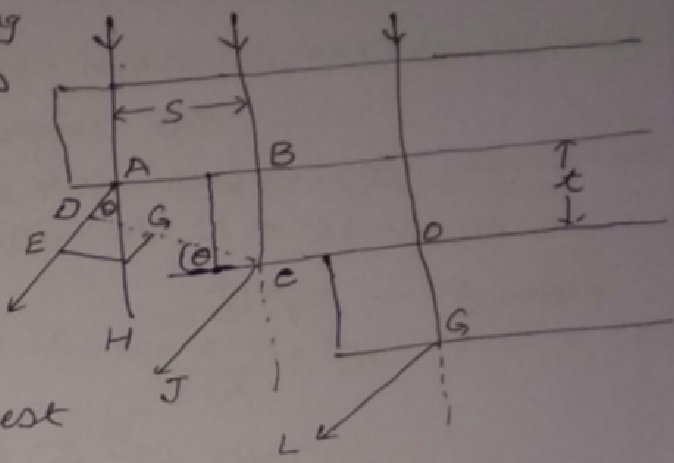
Question :- In what way is an echelon grating superior to an ordinary ruled gratting? Give a short account of its theory and its resolving Power (R.P) and Dispersive Power?

Answer :-

The R.P of a plane grating is $n \times N$. Here n is the order of the spectrum and N is the nos of ruling on the grating surface, with a plane transmission gratting one can see only the second or third order where as with concave grating this order is maximum value upto 6^{th} order. More we ever, if it is very difficult to rule a fine grating upto maximum value of n, under the controlled temp. So we can ~~not~~ see the spectrum of order 10000 and hence this grating has very high R.P

An Echelon grating is shown in figure. It consists of nos of optically waked parallel side glass plates arranged in the form of steps.



Let a parallel beam of light of wavelength λ, fall normally on the largest plate of the echelon. t is the thickness and S is the width of each step.

$$\therefore AB = CO = S$$

Let AH and CJ be the diffracted beam of monochromatic light making an angle θ with the incident beam from the corresponding points A and C. The path difference between the two rays AH and CJ is expressed as path difference

$$= \mu BC - AD$$

$$= \mu BC - (AE - DE)$$

$$= \mu t - (t\cos\theta - S\sin\theta) \quad \text{------------} (1)$$

Here μ is the refractive index of the material of the grating for light of wavelength λ.

So for a principal maximum,

$$\Delta = (\mu - \cos\theta)t + s\sin\theta = n\lambda$$

Here $n = 0, 1, 2, 3 \ldots\ldots$

In real practice θ is small then,

$$\Delta = (\mu - 1)t + s\theta = n\lambda \quad \ldots\ldots (2)$$

for $\mu = 1.52$, $t = 1cm$, $\lambda = 5896 A^\circ$

So in order of the spectrum for $\theta = 0$, along the normal

$$n = \frac{1.52 - 1}{5896 \times 10^{-8}} = 8800$$

This order is very high for the spectrum in comparision with the spectrum produced by plane grating.

RESOLVING POWER :→

Let us consider N plates in the echelon. Then one may have from eq^n (2)

$$N(\mu - 1)t + Ns\theta = Nn\lambda \quad \ldots\ldots (3)$$

which is the condition of principal maxima

The angle of diffraction for the Ist minimum on either side of the principal maximum

$$N(\mu - 1)t + Ns(\theta \pm d\theta) = (nN \pm 1)\lambda \quad \ldots\ldots (4)$$

from (4) — Eq^n (3)

$$d\theta = \frac{\lambda}{Ns} \quad \ldots\ldots (5)$$

This eq^n (5) express half the angular width of the principal maximum of the n^{th} order. Therefore, Rayleigh defined two lines as just resolved when the principal max^m of one falls on the first minimum of the other.

Now, we have for the dispersion from eq^n (2) after diffrentiating w.r to λ.

$$\frac{d\theta}{d\lambda} = \frac{n}{s} - \frac{t}{s}\frac{d\mu}{d\lambda}$$

$$= \frac{(\mu - 1)t}{\lambda s} - \frac{t}{s}\frac{d\mu}{d\lambda} \qquad \left[\because n = (\mu - 1)\frac{t}{\lambda}\right]$$

$$= \frac{t}{s\lambda}\left[(\mu - 1) - \lambda\frac{d\mu}{d\lambda}\right] = \frac{t\mu'}{s\lambda} \quad \ldots\ldots (6)$$

Here $\mu' = (\mu - 1) - \lambda\frac{d\mu}{d\lambda}$

$$\therefore d\theta = \frac{t\mu'}{s\lambda} d\lambda \quad \text{------------} \quad (7)$$

If we equate eqn (5) & (7) for two values of θ, we get

$$\frac{\lambda}{Nsd\lambda} = \frac{t}{s\lambda}\left[(\mu-1) - \lambda\frac{d\mu}{d\lambda}\right]$$

$$\therefore \frac{\lambda}{d\lambda} = \frac{Nt}{\lambda}\left[(\mu-1) - \lambda\frac{d\mu}{d\lambda}\right] \quad \text{------------} \quad (8)$$

This is the expression for the R.P, Again by the use of $t = \frac{n\lambda}{(\mu-1)}$ in eqn (8) we get

$$\frac{\lambda}{d\lambda} = \frac{Nn\lambda}{\lambda(\mu-1)}\left[(\mu-1) - \lambda\frac{d\mu}{d\lambda}\right]$$

$$= \frac{Nn\left[\mu-1 - \lambda\frac{d\mu}{d\lambda}\right]}{(\mu-1)} \quad \text{------------} \quad (9)$$

Since $\frac{d\mu}{d}$ is (−ve), then the effect of dispersion of glass is to increase the R.P from that given by the ordinary grating fro formula $\frac{\lambda}{d\lambda} = Nn$

The R.P resolving limit in the wave number is

$$dv = \frac{dx}{\lambda^2} \quad \text{------------} \quad (10)$$

Now neglecting the (−ve) sign and putting the value of $\frac{\lambda^2}{d\lambda}$ from (8) in (10) we get

$$dv = \frac{1}{Nt\mu'}$$

The value of μ' for Flint glass varies from 0.57 at 6000 A° to 0.73 at 2000 A°. So for the average value taken as 0.66

R.P of for 30 plates $= nN = 8000 \times 30 = 2{,}64000$

Thus is much greater than that of any ruled grating.